**What is a construction?**

Geometric constructions go back to Greek antiquity. They are often called Euclidean constructions, but they certainly predate Euclid. The phrase compass and straightedge construction may be more descriptive. Those are the only instruments allowed. The compass establishes equidistance, and the straightedge establishes collinearity. All geometric constructions are based on those two concepts.

The compass does not simply draw curves. It cannot be replaced by a circle template or a coffee can. The compass is anchored at a center point, and keeps the pencil at a fixed distance from that point. All points on the curve drawn by a compass are equidistant from the center point.

Although rulers are often used as straightedges, the graduation marks may not be used. No measurements are allowed. The straightedge is used only for drawing lines. Given any two distinct points, this instrument can draw the set of all points that are collinear with them. Rulers and protractors have their place in geometry, but these are not construction instruments.

In many commonly accepted constructions (e.g., [congruent angles](http://whistleralley.com/construction/c5.htm)), the compass radius is set by the distance between two points, and then the compass is centered on some third point, elsewhere on the drawing. Under the strictest traditional rules (thank the Greeks again), this was not allowed. The radius was established only by the distance from the center point to some second fixed point. It was assumed that the moment the compass point was lifted, the instrument collapsed, making it impossible to retain the radius.

From: <http://whistleralley.com/construction/whatis.htm>

**Why not use a ruler?**

Why didn't Euclid just measure things with a ruler and calculate lengths? For example, one of the basic constructions is [bisecting a line](http://www.mathopenref.com/constbisectline.html) (dividing it into two equal parts). Why not just measure it with a ruler and divide by two?   
  
The answer is surprising. The Greeks could not do arithmetic. They had only whole numbers, no zero, and no negative numbers. This meant they could not for example divide 5 by 2 and get 2.5, because 2.5 is not a whole number - the only kind they had. Also, their numbers did not use a positional system like ours, with units, tens , hundreds etc, but more like the Roman numerals. In short, they could perform very little useful arithmetic.   
  
So, faced with the problem of finding the midpoint of a line, they could not do the obvious - measure it and divide by two. They had to have other ways, and this lead to the constructions using compass and straightedge. It is also why the straightedge has no markings. It is definitely not a graduated ruler, but simply a pencil guide for making straight lines. Euclid and the Greeks solved problems graphically, by drawing shapes, as a substitute for using arithmetic.

From: <http://www.mathopenref.com/constructions.html>

**Three Impossible Constructions**

The three classical construction problems of antiquity are known as squaring the circle, trisecting an angle, and, doubling a cube. Here is a short description of each of these three problems:

Squaring the Circle: Given a circle, construct a square that has exactly the same area as the circle.

Trisecting an Angle: Given an angle, construct an angle whose measure is exactly 1/3 the measure of the original angle.

Doubling a Cube: Given the length of the side of a (three-dimensional) cube, construct a length so that a cube with an edge of this length will have exactly double the area of the original cube.

From: <http://www.geometer.org/mathcircles/construct.pdf>

Complete Constructions on another sheet of paper. (Do not use the lines on the paper to help you.) Clearly label your work so I know which construction is which.

**Practice each 3 times so you are comfortable with them. I have included a page number, written directions, or a website for all of them. If you like a step-by-step website, you can check out** [**http://www.mathopenref.com/construction.html**](http://www.mathopenref.com/construction.html)

**1.Angle Bisector pg 34**

Why does it work? Think about our triangle congruence theorems.

**2. Segment Bisector/Midpoint pg 33**

**3. Perpendicular bisector (point on line)** (See directions below)

**4. Perpendicular bisector (point not on line)** (See directions below)

**5. Copy an angle pg 34**

**6. Copy a segment pg 33**

**7. Equilateral Triangle (see directions below)**

How do we know it is an equilateral triangle?

How do we know each angle is 60º?

**8. Parallel lines pg 152**

**9. Square watch http://www.youtube.com/watch?v=nmFwr9aNf3w**

**10. Regular hexagon inscribed in a circle watch** [**http://www.mathopenref.com/consthexagon.html**](http://www.mathopenref.com/consthexagon.html)

***Given point P on line k, construct a line through P, perpendicular to k.***

|  |  |
| --- | --- |
| 1. Begin with line *k*, containing point *P*. | http://whistleralley.com/construction/c2s1.gif |
| 2. Place the compass on point *P*. Using an arbitrary radius, draw arcs intersecting line *k* at two points. Label the intersection points *X* and *Y*. | http://whistleralley.com/construction/c2s2.gif |
| 3. Place the compass at point *X*. Adjust the compass radius so that it is more than (1/2)*XY*. Draw an arc as shown here. | http://whistleralley.com/construction/c2s3.gif |
| 4. Without changing the compass radius, place the compass on point *Y*. Draw an arc intersecting the previously drawn arc. Label the intersection point *A*. | http://whistleralley.com/construction/c2s4.gif |
| 5. Use the straightedge to draw line *AP*. Line *AP* is perpendicular to line *k*. | http://whistleralley.com/construction/c2s5.gif |

***Given point R, not on line k, construct a line through R, perpendicular to k.***

|  |  |
| --- | --- |
| 1. Begin with point line *k* and point *R*, not on the line. | http://whistleralley.com/construction/c3s1.gif |
| 2. Place the compass on point *R*. Using an arbitrary radius, draw arcs intersecting line *k* at two points. Label the intersection points *X* and *Y*. | http://whistleralley.com/construction/c3s2.gif |
| 3. Place the compass at point *X*. Adjust the compass radius so that it is more than (1/2)*XY*. Draw an arc as shown here. | http://whistleralley.com/construction/c3s3.gif |
| 4. Without changing the compass radius, place the compass on point *Y*. Draw an arc intersecting the previously drawn arc. Label the intersection point *B*. | http://whistleralley.com/construction/c3s4.gif |
| 5. Use the straightedge to draw line *RB*. Line *RB* is perpendicular to line *k*. | http://whistleralley.com/construction/c3s5.gif |

***Given a line segment as one side, construct an equilateral triangle.***

***This method may also be used to construct a 60 angle.***

|  |  |
| --- | --- |
| 1. Begin with line segment *TU*. | c7s1 |
| 2. Center the compass at point *T*, and set the compass radius to *TU*. Draw an arc as shown. | c7s2 |
| 3. Keeping the same radius, center the compass at point *U* and draw another arc intersecting the first one. Let point *V* be the point of intersection. | c7s3 |
| 4. Draw line segments *TV* and *UV*. Triangle *TUV* is an equilateral triangle, and each of its interior angles has a measure of 60. | c7s4 |